On the nonlinear behaviour of reinforced concrete frames

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An experimental investigation is described in which a large-scale reinforced concrete plane frame is tested to study factors contributing to its nonlinear behaviour under short-term loading conditions. The test results indicate that frame behaviour can be significantly affected by second-order influences such as material nonlinearities, geometric nonlinearities, concrete shrinkage, tension stiffening effects, shear deformations, and membrane action. A nonlinear frame analysis procedure, previously developed taking these mechanisms into account, is shown to accurately predict most aspects of behaviour, including deflection response, ultimate load capacity, and failure mechansim. Aspects of the theoretical modelling which are in need of further improvement are also identified.

Key words: analysis, behaviour, deformation, frame, large scale, nonlinear, reinforced concrete, strength, test.

Cet article décrit une étude expérimentale au cours de laquelle une ossature plane en béton armé pleine grandeur a été soumise à des essais en vue d'étudier les facteurs qui contribuent au comportement non linéaire lorsque l'ossature est soumise à court terme à des charges monotoniques. Les résultats des essais indiquent que le comportement de l'ossature peut être influencé par des éléments de second ordre comme la non-linéarité des matériaux, les nonlinéarités géométriques, le retrait du béton, les effets du raidissement par la traction, les déformations causées par le cisaillement et le mouvement de la membrane. Une méthode d'analyse non linéaire de l'ossature, élaborée précédemment afin de tenir compte de ces mécanismes, permet de prédire avec précision la plupart des caractéristiques du comportement, y compris le comportement en déformation, la capacité de charge ultime et les mécanismes de défaillance. Des aspects de la modélisation théorique qui nécessitent des améliorations sont également énumeres.

Mots clés : analyse, comportement, flèche, ossature, grande échelle, non linéaire, béton armé, résistance, essai.

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1. Introduction

It is generally well understood that reinforced concrete elements exhibit highly nonlinear load–deformation response, due primarily to the nonlinear stress–strain behaviour of the constituent materials. Still, it is common practice to design reinforced concrete frame systems based on the member forces determined from linear elastic structural analyses. In most instances, this approach leads to a satisfactory design. Situations can arise, however, where a more comprehensive appraisal of frame response is desired; for example, when accurate estimates are needed for service load deflections or for ultimate load capacity (see Fig. 1). Also, occasions arise in applying code design procedures where a more thoughtful consideration of response can be effectively utilized. In the Canadian code (CSA 1984) this occurs, for example, when considering member design forces (Clause 8.3.3), deflections (Clause 9.5.2.3), or slenderness effects in columns (Clause 10.10.1). In such cases, the nature of the nonlinear response of a frame, and the factors contributing to it, become of interest.

Various nonlinear frame analysis procedures have been developed in recent years (e.g., Diaz and Roesset 1987; Mo 1986; Wong *et al.* 1987) which capture, to varying degrees, the complex behaviour of reinforced concrete. Among the methods proposed is that by Vecchio (1987), which places particular importance on accurately modelling both material and structural behaviour. Deficient from the description of most analytical procedures, however, is a corroboration with experimentally observed behaviour. This failing is primarily due to the sparsity of suitable test data found in the literature. While some useful data are available from tests conducted on subassemblies or small-scale frames (e.g., Rad and Furlong 1980), the more valuable data derived from larger scale structures are lacking.

In response to this perceived need, an experimental investigation of limited scope was undertaken. From a simple well-defined test on a large-scale frame model, the objectives were to observe the factors influencing the response of the frame and to examine current abilities to predict this response. The details and findings of this study are discussed herein.

2. Test program

The test model chosen for investigation was a large-scale one-span, two-storey plane frame as shown in Fig. 2. The frame was designed with a centre-to-centre span of 3500 mm, a storey height of 2000 mm, and an overall height of 4600 mm. All frame members were 300 mm wide by 400 mm deep. The frame was built integral with a large, heavily reinforced concrete base.

The columns and the end regions of the first-storey beam were similarly reinforced with four No. 20 deformed bars as bottom reinforcement, four No. 20 bars as top reinforcement,

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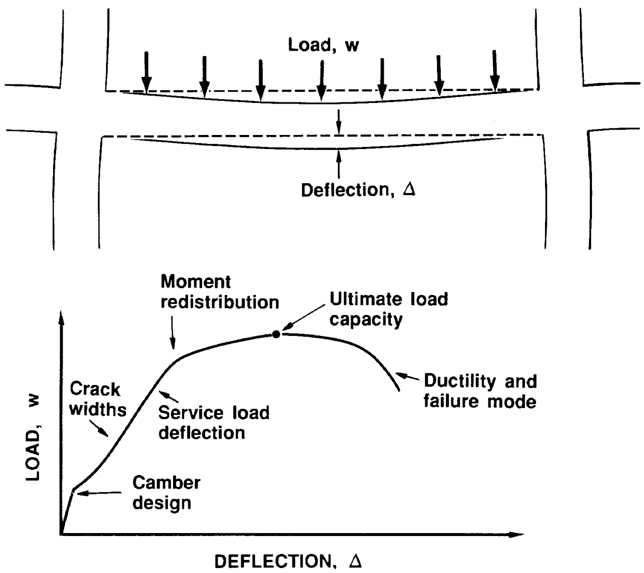

FIG. 1. Typical frame behaviour characteristics influenced by nonlinear response.

and No. 10 closed stirrups at 125 mm spacing as shear reinforcement. In the central 500 mm length of the first-storey beam, the top reinforcement was cut back to two No. 20 bars. The second-storey beam was more lightly reinforced, with three No. 20 bars top and bottom, and No. 10 stirrups at 250 mm spacing. At all locations, placement of the reinforcement was such as to provide a clear cover of 30 mm. Section details are given in Fig. 2. Note that the longitudinal reinforcement was anchored at member ends by welding the bars to stiff bearing plates.

The frame was formed and cast in a reclined position, using a low-slump ready-mix concrete with superplasticizer added. After curing for 25 days, the formwork was stripped and the frame was lifted to its upright position. The base of the frame was then bolted to the laboratory strong-floor, giving an essentially fully-fixed condition at the base.

The concrete material properties were determined on the day of testing from 150 × 300 mm standard test cylinders. The cylinder tests were conducted in a 5000 kN capacity MTS ''stiff'' testing machine, in stroke control mode, using a stroke rate of 6.67×10^{-3} mm/s. The concrete was found to have a compressive strength of 29 MPa and a stress–strain curve as shown in Fig. 3*a*.

The material properties of the No. 20 rebar, used as longitudinal reinforcement in all members, were determined from coupons tested in a 100 kN MTS testing machine. Based on a nominal cross-sectional area of 300 mm^2, the rebar steel was found to have a yield stress of 418 MPa, an ultimate stress of 596 MPa, and a modulus of elasticity of 192 600 MPa. A typical stress–strain curve is shown in Fig. 3*b*. Note that the bars exhibited a rather short yield plateau, reaching a strain of between 7.5×10^{-3} and 11.0×10^{-3} mm/mm. The strain hardening modulus observed thereafter had an average value of 3100 MPa. The No. 10 bars used for shear reinforcement had a yield strength of 454 MPa and an ultimate stress of 640 MPa.

Testing of the frame model was to involve the simple case of monotonically increasing point load applied at the midspan of the first-storey beam. The test setup devised accordingly is illustrated in Fig. 4. A steel I-beam was placed across the first-storey beam, at the midspan, bearing on a neoprene pad of 300 mm width. Load was then applied using two 350 kN capacity servo-controlled actuators, symmetrically placed on both sides of the frame, pulling down on the transverse beam. The actuators, in turn, were mounted to steel A-frames bolted to the laboratory strong-floor.

To monitor behaviour during testing, the frame model was extensively equipped with electronic instrumentation. Twelve displacement transducers (LVDTs), positioned at key points around the frame, were used to monitor deformations. Measurements of surface strain were made using electronic ''Zurich'' gauges on 200 mm gauge lengths along each member. (Longitudinal strains at the levels of the top and bottom reinforcement were monitored; no transverse or diagonal strain readings were taken.) In addition, 22 electrical strain gauges were used to measure strains in the reinforcement at various locations. Load cells integral with the load actuators provided a measure of the applied load. Continuous monitoring and recording of data was accomplished using a computer-controlled scanning system.

Load was applied to the test frame with the actuators in a stroke control mode, using an average stroke rate of 2.0×10^{-3} mm/s. Special attention was given to maintaining, at all times, the same stroke in both actuators. Discrete load stages were defined whereat loading (i.e., stroke) was held constant while crack patterns were recorded and visual inspection of the frame was made. The load stages were initially set at approximately 15 kN load increments, reduced to 10 kN at intermediate stages of the test. In the late stages of loading, the load stages were defined by stroke increments of 2.5 mm.

Testing continued over several days, with a total of 36 load stages recorded. At the end of each day, the frame was unloaded and residual conditions were recorded. The following day, the initial load stage would be at a load level approximately equal to the final load achieved the previous day.

3. Test observations

The test frame had experienced appreciable shrinkage cracking by the end of the curing period (i.e, 25 days). With the frame still in the reclined position, the top surface of each member was observed to contain an average of six cracks of 0.20–0.25 mm width, uniformly distributed along the length of the member. The cracks were typically full-width, but did not extend deeper than approximately 100 mm. Few shrinkage cracks developed on the bottom surface, nor did the cracks on the top surface propagate further during lifting of the frame or in the remaining time up to testing.

Testing of the frame began approximately 6 months after casting. As discussed, a gradually increasing monotonic point load was applied to the first-storey beam at the midspan. The test history, in terms of a load–deformation response measured by actuator stroke, is shown in Fig. 5. Key load stages are identified.

Distinct flexural cracks appeared at the bottom midspan region of the beam at a load of 45 kN (load stage LS3). However, no corresponding change was seen in the stiffness of the load–deformation response at this time. As loading continued, flexural cracking developed at the joint regions of the beam and in the lower portions of the first-storey columns. Flexural-shear cracks also soon began to develop in the intermediate regions of the loaded beam. By a load of 120 kN

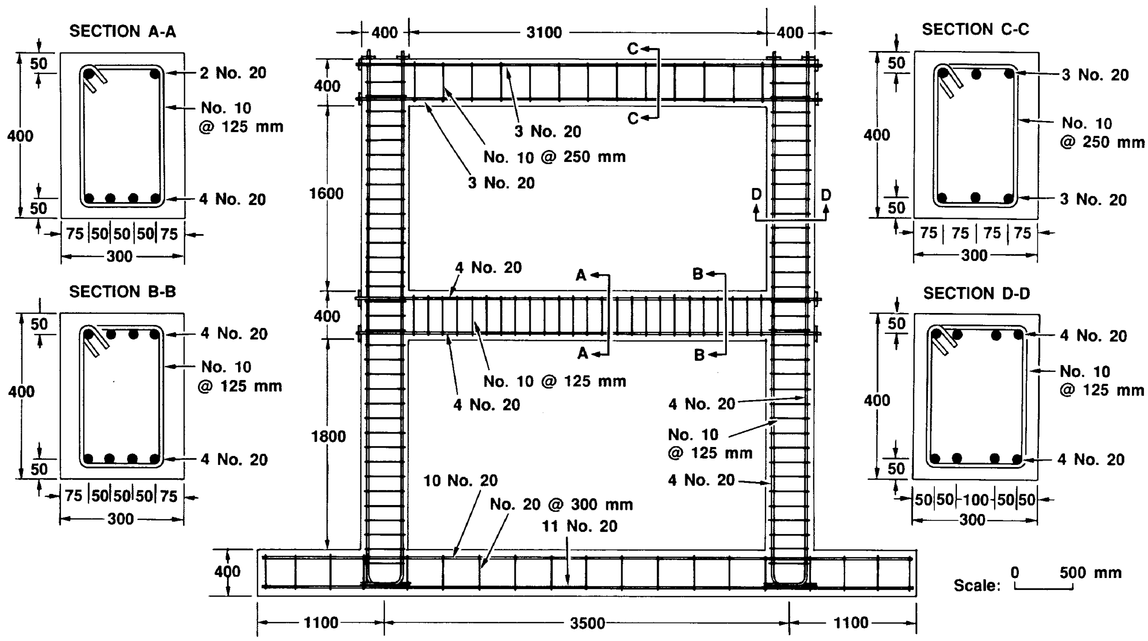

FIG. 2. Details of test frame.

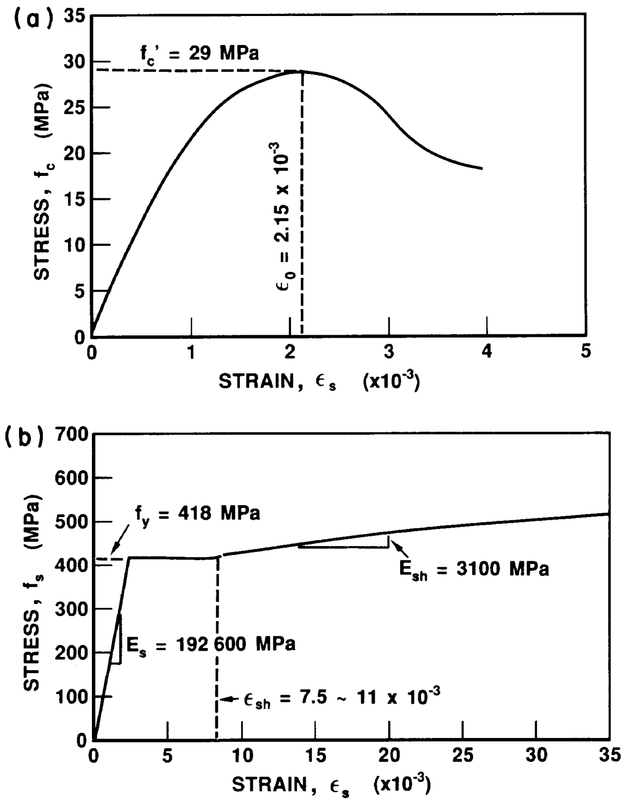

FIG. 3. Material properties: (*a*) concrete; (*b*) longitudinal reinforcement.

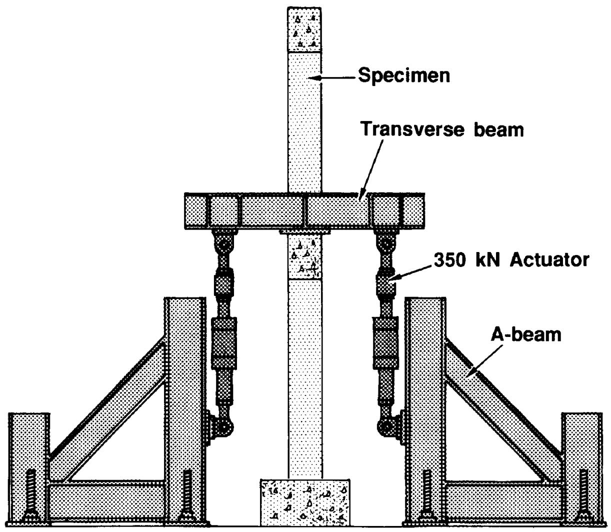

FIG. 4. Test setup.

(LS8), the crack pattern in the beam was well established and a perceptible change in stiffness was observed. Thereafter, the effect of additional load was mostly to increase the width of existing cracks rather than to cause the formation of new cracks. The progression of cracking in the columns was much more gradual, with flexural cracks still emerging as load increased beyond 300 kN. In general, crack widths at this stage of loading were limited to 0.75 mm in the beam and 0.25 in the columns.

At a load of 360 kN (LS24), the bottom reinforcement in the beam, at the midspan, was deduced from strain readings to have yielded. This led to dramatic increases in crack widths (exceeding 1.00 mm) and a further decline in the stiffness of the load–deformation response. Shortly after, at 370 kN load, the top reinforcement at both ends of the beam yielded. Thereafter, further deformation of the beam was concentrated at the rapidly widening cracks at the midspan and joint regions.

Evidence of concrete crushing and yielding of both the top and the bottom bars suggested the ultimate capacity of the beam section, at the midspan, had been attained at a load of

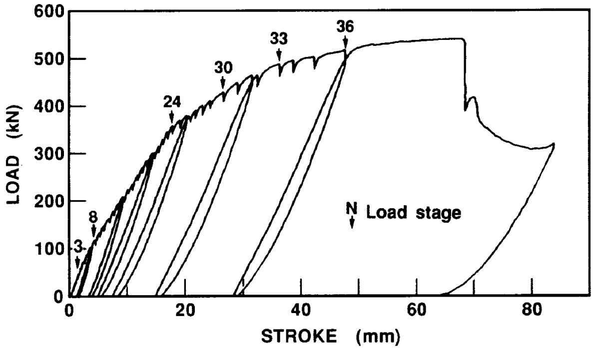

FIG. 5. Test history in terms of actuator load-stroke.

430 kN (LS30). Hence, a plastic hinge had formed at the midspan, resulting in redistribution of moments upon further loading. At 489 kN (LS33), crushing of concrete and accelerated straining of the reinforcement suggested that the beam section at the beam–column joints was approaching ultimate capacity. A three-hinge failure mechanism could be seen developing.

An ultimate load of 517 kN (LS36) was achieved before the frame structure was deemed to have exhausted its capacity. Although severe local crushing of the concrete and widespread yielding of reinforcement had occurred, the frame exhibited a high degree of ductility and was able to maintain ultimate load. To observe the final failure mode, the frame was unloaded and then reloaded at a much faster stroke rate (50.0×10^{-3} mm/s). The increased loading rate resulted in a seemingly higher ultimate load than previously achieved, reaching 540 kN before sudden failure occurred. The failure mode involved the combination of a flexural collapse mechanism (plastic hinges at the midspan and joints) and a shear failure near the beam midspan. A view of the frame, after failure, is given in Fig. 6.

Various aspects of the response of the frame are illustrated in Fig. 7. Beam deflections are given in Fig. 7*a*, beam and column elongations are described in Fig. 7*b*, and rebar strains at selected locations are given in Fig. 7*c*.

4. Discussion of frame behaviour

Test observations indicated that a number of second-order effects had a significant influence on the nonlinear behaviour of the frame. Apparent in the observed response were the influences of concrete shrinkage, concrete tension stiffening, shear deformation, membrane action, material nonlinearities, and geometric nonlinearities.

Drying-related shrinkage of concrete, in a statically indeterminate or heavily reinforced structure, can significantly affect response during early stages of loading. Restrained drying shrinkage in the frame tested resulted in the presence of high initial tensile stresses in the concrete and in the formation of several cracks along each member, particularly in the joint regions. These initial stresses and cracks combined to produce a much lower stiffness in pre-cracking load–deformation response than had been anticipated. In addition, first flexural cracking was observed at a lower load than expected, but without noticeable variation between pre- and post-cracking stiffness.

Concrete tension stiffening refers to the continued presence of tensile stresses in concrete, between cracks, and to the resulting influence on the post-cracking load–deformation response of the structure. To ignore this effect by assuming that tensile stresses in the concrete are zero after cracking, as is commonly done, typically results in a significant overestimation of deformation at service load levels. Tension stiffening effects, as evidenced in the test frame, caused a gradual transition to occur between uncracked and fully cracked member stiffnesses (i.e., between I_g and I_{cr}) and produced a gradually softening load–deformation response.

The material nonlinearities of concrete and reinforcement obviously have a large influence on the behaviour of any reinforced concrete structure. In the test conducted, the effects of yielding of the reinforcement were most noticeable, resulting in immediately apparent changes in member stiffness and in a concentration of deformation and damage in local regions (i.e., hinges located at the beam midspan and end joints). Strain hardening in the reinforcement contributed to a slowly increasing ultimate load capacity even after a three-hinge collapse mechanism had formed. Ultimately, the ductility of the load–deformation response was limited by a brittle crushing of the concrete.

Geometric nonlinearities can influence deformations or ultimate load capacity under conditions where *P*-delta or moment magnification effects are significant. In this test, the effects of geometric nonlinearities were negligible.

A typical reinforced concrete section subjected to bending will develop flexural cracks, resulting in the average strains on the tension face being much larger in magnitude than the compressive strains on the opposite face. Thus, the average strain in the section (i.e., at the mid-depth) is tensile which, when integrated over the length of the member, results in a net elongation. However, when the flexural member is restrained from freely elongating, as in this case by the columns, an axial compressive force is induced in the member. The axial compression, in turn, stiffens the response of the member and increases its moment capacity. This mechanism, known as membrane action (see Fig. 8), was particularly significant in the observed response of the test frame. The ultimate load capacity was increased by 25% from 412 kN, which would be predicted based on a simple three-hinge mechanism using the nominal pure moment capacities of the beam sections (i.e., no axial load, no material resistance factors). (Uncertainties regarding the degree of restraint offered by the columns and the manifestation of membrane action were a prime motivation in performing the frame test.)

Finally, in most analyses of reinforced concrete frame response, it is common and appropriate to ignore the effects of shear deformation. In this case, however, shear may be responsible for a significant portion of the total deformation observed because of the relatively high depth-to-span ratios of the frame members. A direct measure of the shear deformation could not be made, since shear strain data were not collected. However, differences between the total displacements (measured by displacement transducers on the beam) and flexural displacements (estimated from curvatures computed from longitudinal strains measured along the top and bottom of the beam) indicated shear deformations to be significant (from 10 to 30% of total). Also, shear behaviour was a significant factor in the final failure mode of the frame.

5. Theoretical predictions of response

Theoretical predictions of the frame response were obtained

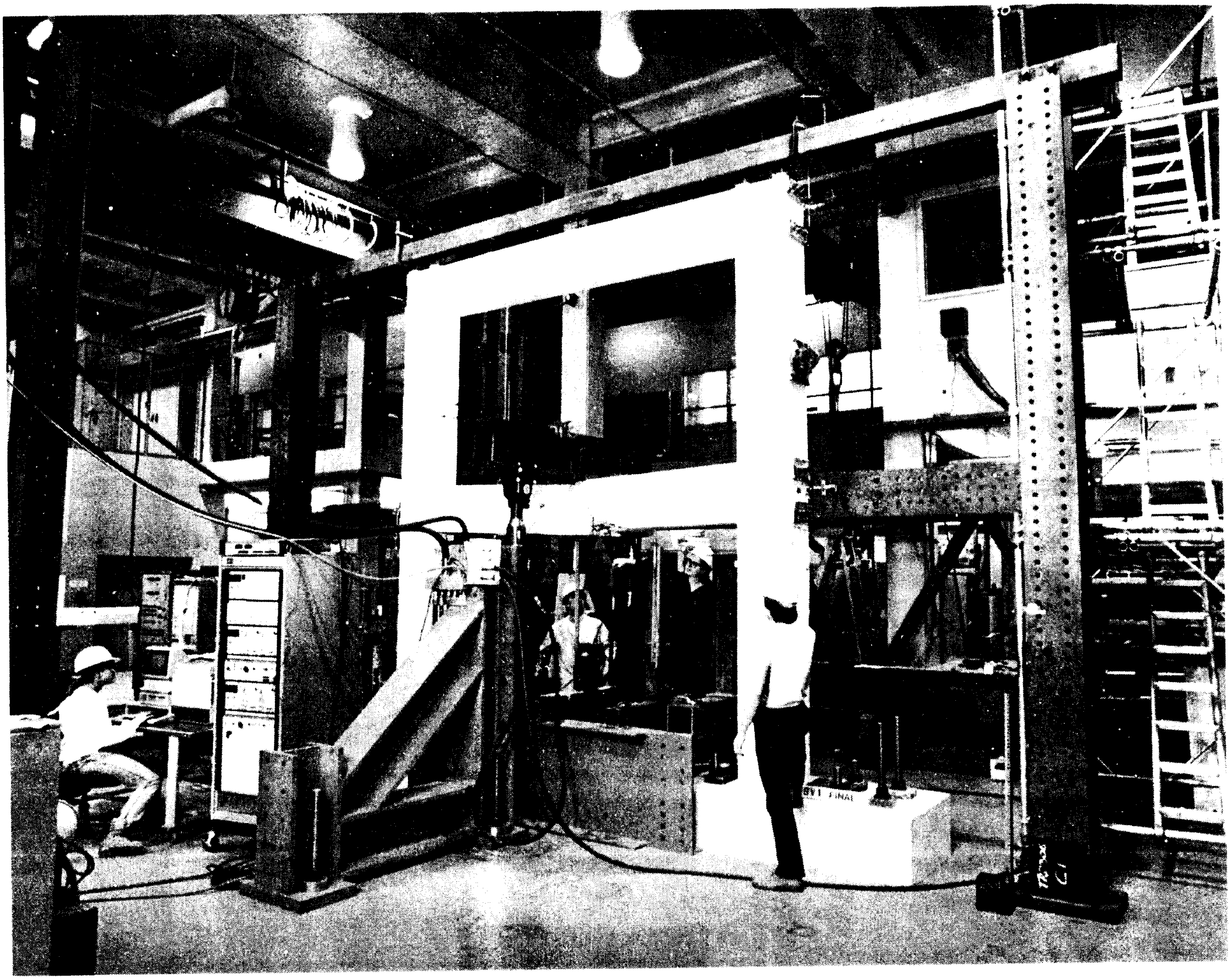

FIG. 6. View of test frame after failure.

using TEMPEST (Vecchio 1987), a program for nonlinear analysis of reinforced concrete plane frames subjected to mechanical and (or) thermal loads. The program inherently takes into account second-order effects such as nonlinear constitutive response of the concrete and reinforcement, tension and stiffening effects, membrane action, geometric nonlinearity, load history, creep, shrinkage, and variable member geometry and reinforcement. It currently does not, however, make allowances for shear effects.

To include a consideration of shear effects, program SMAL (Vecchio and Collins 1988) was utilized. This program computes the response of beam sections to combined shear, moment, and axial loads according to the formulations of the modified compression field theory (Vecchio and Collins 1986). Thus, the shear deformation response of the frame members were analyzed independently and then superimposed onto the results obtained from TEMPEST.

In performing an analysis of the test frame, a computer model of one-half of the structure was created consistent with the frame's geometry and reinforcement details. Eighteen member segments were used in modelling the test frame. The material properties used were as determined from the cylinder and coupon tests. A uniform concrete shrinkage strain of −0.0005 mm/mm was also used, estimated from the number and width of the shrinkage cracks observed on the frame before testing.

Comparisons of some aspects of the predicted and observed responses of the test frame are made in Fig. 9. Figure 9*a* compares the load−deflection response of the first-storey beam at the midspan. The gradually softening response is simulated reasonably well, although the theoretical response tends to be somewhat stiffer at intermediate load levels. The ultimate load capacity is accurately predicted, however, with the computed value of 510 kN corresponding to 98% of the observed strength. Not apparent in the response curves is the fact that aspects of local behaviour (e.g., crack progression, yielding, hinging) are also reasonably well predicted. For example, TEMPEST predicts the formation of the first plastic hinge at 425 kN; experimental data suggest that it occured at 429 kN.

Elongation of the first-storey beam is shown in Fig. 9*b*. Again, while providing a reasonable estimate of the observed response, the theoretical predictions appear to be too stiff. Note that near ultimate load, the beam elongation approaches 5.0 mm, a value several times larger than would be predicted from a linear elastic analysis. Strains in the bottom reinforcement of the beam, at the midspan, are compared in Fig. 9*c*.

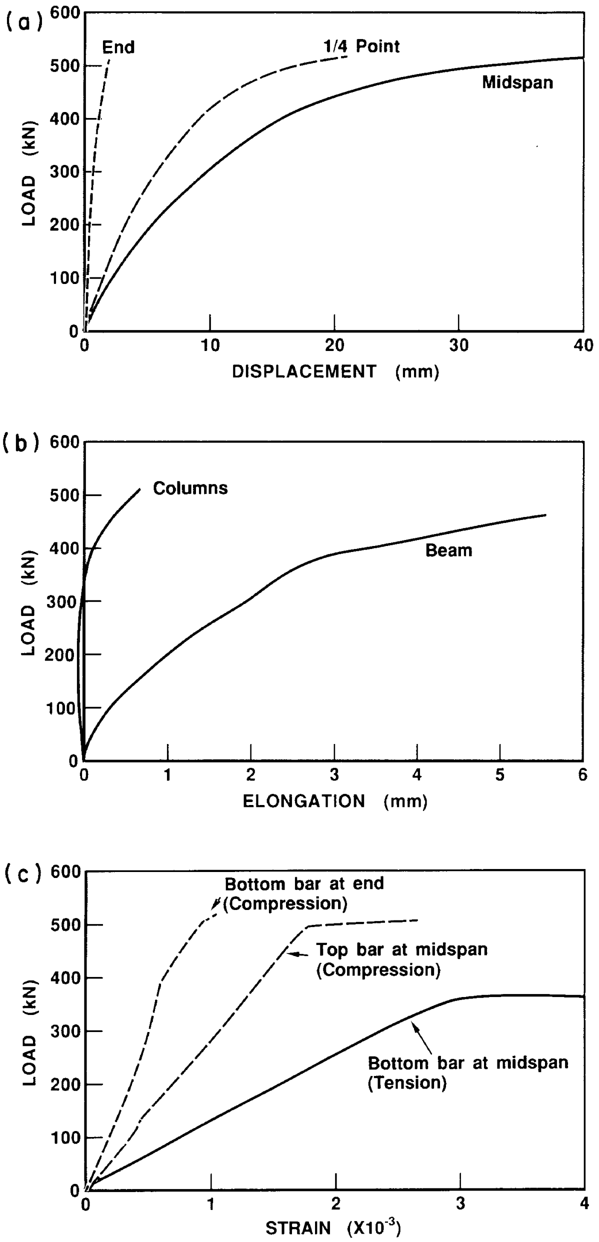

FIG. 7. Measured response of test frame: (*a*) beam deflections; (*b*) beam and column elongation; (*c*) beam reinforcement strains.

Again, reasonably good agreement exists between the predicted and the observed response.

The slight inaccuracies evident in the predicted responses may be arising from several sources. First, the tension stiffening formulation used is based on the data derived from membrane element tests (Vecchio and Collins 1986). This formulation is recognized as being generally too stiff for applications involving flexural members. Further work is required to develop more appropriate constitutive models in this regard. Second, the effects of shear were only roughly approximated using a decoupled sectional analysis. Interactions between shear, moment, and axial loads, and their influence on overall frame response, would be better considered by incorporating into TEMPEST appropriate shear behaviour models. Work in this regard is currently underway. Lastly, the shrinkage strain assumed in the analysis was merely a rough estimate, averaged over the entire specimen. More appropriate would be a time-dependent formulation taking into account the influence of size, curing, mix proportions, location, and other factors. Such a formulation could easily be accommodated into the analysis program.

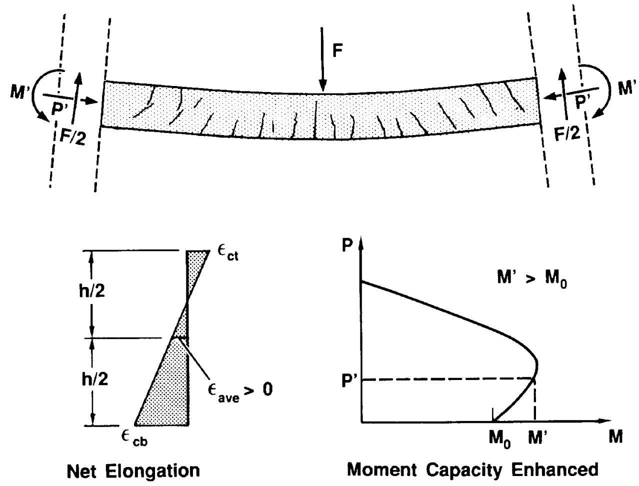

FIG. 8. Membrane action contributing to enhancement of load capacity.

Several factors have been identified as contributing to the nonlinear behaviour of the test frame. It would be of interest to determine what relative degree of influence each of these factors has on the overall response. Shown in Fig. 10 are predictions of the frame's load–deformation response, indicating the degree of influence arising separately from material nonlinearity, membrane action, shrinkage, tension stiffening, and shear deformation. The response curve for the case of material nonlinearity not being considered is based on an assumed stiffness of $0.5I_g$ for each member. The ultimate load cut-off occurs at the theoretical plastic limit, ignoring the beneficial effects of axial compression arising from membrane action. In general, deformations and ultimate capacity are both significantly underestimated. Ignoring shrinkage effects results in a considerably higher initial stiffness than would be otherwise predicted; ignoring tension stiffening effects would have the reverse effect. In neither case is the ultimate capacity affected significantly. The consequences of ignoring shear effects are seen to increase with load, resulting in about a 20% reduction in computed deflection. Finally, it was found that ignoring geometry nonlinearity had very little effect in this particular instance.

6. Conclusions

A large-scale reinforced concrete frame was tested under a simple monotonic loading condition, and aspects pertaining to the frame's nonlinear response were observed and studied. The test data obtained were used to investigate the accuracy of an analytical procedure previously developed.

The test results indicate that several influencing factors should be considered when attempting to make an accurate prediction of the deformation or ultimate load capacity of a reinforced concrete frame. Nonlinearities in the constitutive response of concrete and reinforcement, of course, remain as the major influence at all stages of loading. Concrete shrinkage effects can be prominent in the pre-cracking load–deformation response, resulting in a behaviour much softer than

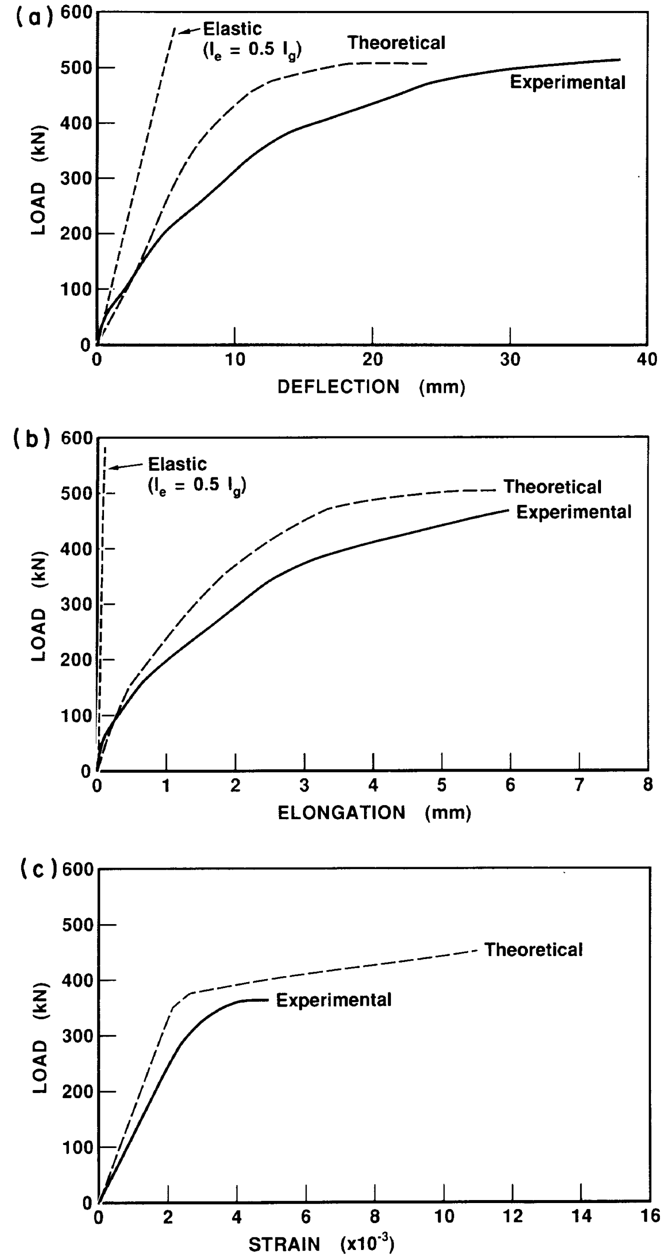

FIG. 9. Comparison of predicted and observed response: (*a*) deflection of beam at midspan; (*b*) elongation of beam; (*c*) beam bottom rebar strain at midspan.

would be predicted using uncracked section properties. After cracking, tension stiffening effects in the concrete will result in a gradual rather than sudden decrease in member stiffness. As a result, frame deflections at service load levels will be significantly less than would be predicted using fully cracked section properties. At ultimate, membrane action can result in a substantial increase in load capacity, as compared to that predicted using the nominal flexural capacities of the sections. In frames having members with relatively high depth-to-span ratios, shear effects can also contribute significantly to the

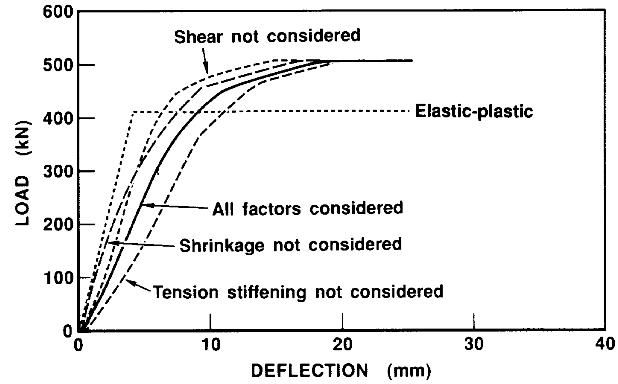

FIG. 10. Influence of various factors on theoretical response.

deformation or failure mode of the structure. Although not significant in this study, the effects of geometry nonlinearity should also be considered. As well, under long-term load condition, creep effects can be a major factor.

The complex behaviour of reinforced concrete frames can be accurately predicted using appropriate nonlinear analysis procedures, provided the influencing factors described are adequately modelled. The procedure described herein exhibited reasonably good accuracy in predicting deflections, load capacity, and the failure mechanisms. Improvements could be achieved, however, by developing a more refined tension stiffening model and by incorporating shear effects into the analysis procedure. Work is currently progressing in this regard.

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