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Compression Field Modeling of Reinforced Concrete Subjected to Reversed Loading: Formulation

by Daniel Palermo and Frank J. Vecchio

Constitutive formulations are presented for concrete subjected to reversed cyclic loading consistent with a compression field approach. The proposed models are intended to provide substantial compatibility to nonlinear finite element analysis in the context of smeared rotating cracks in both the compression and tension stress regimes. The formulations are also easily adaptable to a fixed crack approach or an algorithm based on fixed principal stress directions. Features of the modeling include: nonlinear unloading using a Ramberg-Osgood formulation; linear reloading that incorporates degradation in the reloading stiffness based on the amount of strain recovered during the unloading phase; and improved plastic offset formulations. Backbone curves from which unloading paths originate and on which reloading paths terminate are represented by the monotonic response curves and account for compression softening and tension stiffening in the compression and tension regions, respectively. Also presented are formulations for partial unloading and partial reloading.

Keywords: cracks; load; reinforced concrete.

RESEARCH SIGNIFICANCE

The need for improved methods of analysis and modeling of concrete subjected to reversed loading has been brought to the fore by the seismic shear wall competition conducted by the Nuclear Power Engineering Corporation of Japan.[1] The results indicate that a method for predicting the peak strength of structural walls is not well established. More important, in the case of seismic analysis, was the apparent inability to accurately predict structure ductility. Therefore, the state of the art in analytical modeling of concrete subjected to general loading conditions requires improvement if the seismic response and ultimate strength of structures are to be evaluated with sufficient confidence.

This paper presents a unified approach to constitutive modeling of reinforced concrete that can be implemented into finite element analysis procedures to provide accurate simulations of concrete structures subjected to reversed loading. Improved analysis and design can be achieved by modeling the main features of the hysteresis behavior of concrete and by addressing concrete in tension.

INTRODUCTION

The analysis of reinforced concrete structures subjected to general loading conditions requires realistic constitutive models and analytical procedures to produce reasonably accurate simulations of behavior. However, models reported that have demonstrated successful results under reversed cyclic loading are less common than models applicable to monotonic loading. The smeared crack approach tends to be the most favored as documented by, among others, Okamura and Maekawa[2] and Sittipunt and Wood.[3] Their approach, assuming fixed cracks, has demonstrated good correlation to experimental results; however, the fixed crack assumption requires separate formulations to model the normal stress and shear stress hysteretic behavior. This is at odds with test observations. An alternative method of analysis, used herein, for reversed cyclic loading assumes smeared rotating cracks consistent with a compression field approach. In the finite element method of analysis, this approach is coupled with a secant stiffness formulation, which is marked by excellent convergence and numerical stability. Furthermore, the rotating crack model eliminates the need to model normal stresses and shear stresses separately. The procedure has demonstrated excellent correlation to experimental data for structures subjected to monotonic loading.[4] More recently, the secant stiffness method has successfully modeled the response of structures subjected to reversed cyclic loading,[5] addressing the criticism that it cannot be effectively used to model general loading conditions.

While several cyclic models for concrete, including Okamura and Maekawa;[2] Mander, Priestley, and Park;[6] and Mansour, Lee, and Hsu,[7] among others, have been documented in the literature, most are not applicable to the alternative method of analysis used by the authors.

Documented herein are models, formulated in the context of smeared rotating cracks, for reinforced concrete subjected to reversed cyclic loading. To reproduce accurate simulations of structural behavior, the modeling considers the shape of the unloading and reloading curves of concrete to capture the energy dissipation and the damage of the material due to load cycling. Partial unloading/reloading is also considered, as structural components may partially unload and then partially reload during a seismic event. The modeling is not limited to the compressive regime alone, as the tensile behavior also plays a key role in the overall response of reinforced concrete structures. A comprehensive review of cyclic models available in the literature and those reported herein can be found elsewhere.[8]

It is important to note that the models presented are not intended for fatigue analysis and are best suited for a limited number of excursions to a displacement level. Further, the models are derived from tests under quasistatic loading.

CONCRETE STRESS-STRAIN MODELS

For demonstrative purposes, Vecchio[5] initially adopted simple linear unloading/reloading rules for concrete. The formulations were implemented into a secant stiffness-based finite element algorithm, using a smeared rotating crack

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approach, to illustrate the analysis capability for arbitrary loading conditions, including reversed cyclic loading. The models presented herein have also been formulated in the context of smeared rotating cracks, and are intended to build upon the preliminary constitutive formulations presented by Vecchio.[5] A companion paper[9] documenting the results of nonlinear finite element analyses, incorporating the proposed models, will demonstrate accurate simulations of structural behavior.

Compression response

First consider the compression response, illustrated in Fig. 1, occurring in either of the principal strain directions. Figure 1(a) and (b) illustrate the compressive unloading and compressive reloading responses, respectively. The backbone curve typically follows the monotonic response, that is, Hognestad parabola[10] or Popovics formulation,[11] and includes the compression softening effects according to the Modified Compression Field Theory.[12]

The shape and slope of the unloading and reloading responses are dependent on the plastic offset strain ε_c^p, which is essentially the amount of nonrecoverable damage resulting from crushing of the concrete, internal cracking, and compressing of internal voids. The plastic offset is used as a parameter in defining the unloading path and in determining the degree of damage in the concrete due to cycling. Further, the backbone curve for the tension response is shifted such that its origin coincides with the compressive plastic offset strain.

Various plastic offset models for concrete in compression have been documented in the literature. Karsan and Jirsa[13] were the first to report a plastic offset formulation for concrete subjected to cyclic compressive loading. The model illustrated the dependence of the plastic offset strain on the strain at the onset of unloading from the backbone curve. A review of various formulations in the literature reveals that, for the most part, the models best suit the data from which they were derived, and no one model seems to be most appropriate. A unified model (refer to Fig. 2) has been derived herein considering data from unconfined tests from Bahn and Hsu[14] and Karsan and Jirsa,[13] and confined tests from Buyukozturk and Tseng.[15] From the latter tests, the results indicated that the plastic offset was not affected by confining stresses or strains. The proposed plastic offset formulation is described as

$$\varepsilon_c^p = \varepsilon_p\left[0.166\left(\frac{\varepsilon_{2c}}{\varepsilon_p}\right)^2 + 0.132\left(\frac{\varepsilon_{2c}}{\varepsilon_p}\right)\right] \quad (1)$$

where ε_c^p is the plastic offset strain; ε_p is the strain at peak stress; and ε_{2c} is the strain at the onset of unloading from the backbone curve. Figure 2 also illustrates the response of other plastic offset models available in the literature.

The plot indicates that models proposed by Buyukozturk and Tseng[15] and Karsan and Jirsa[13] represent upper- and lower-bound solutions, respectively. The proposed model (Palermo) predicts slightly larger residual strains than the lower limit, and the Bahn and Hsu[14] model calculates progressively larger plastic offsets. Approximately 50% of the datum points were obtained from the experimental results of Karsan and Jirsa;[13] therefore, it is not unexpected that the Palermo model is skewed towards the lower-bound Karsan and Jirsa[13] model. The models reported in the literature were derived from their own set of experimental data and, thus, may be affected by the testing conditions. The proposed formulation alleviates dependence on one set of experimental data and test conditions. The Palermo model, by predicting

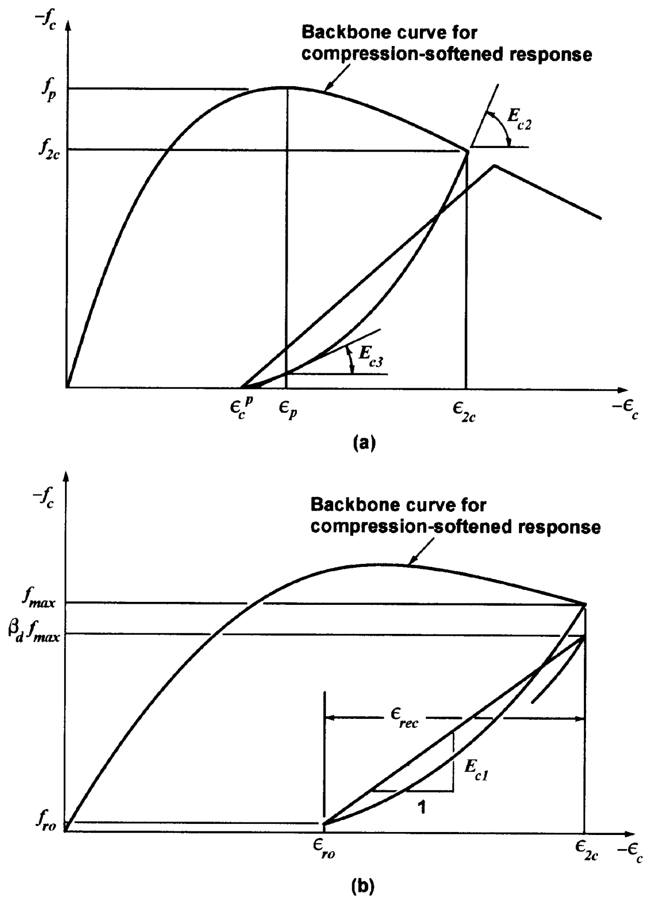

Fig. 1—Hysteresis models for concrete in compression: (a) unloading; and (b) reloading.

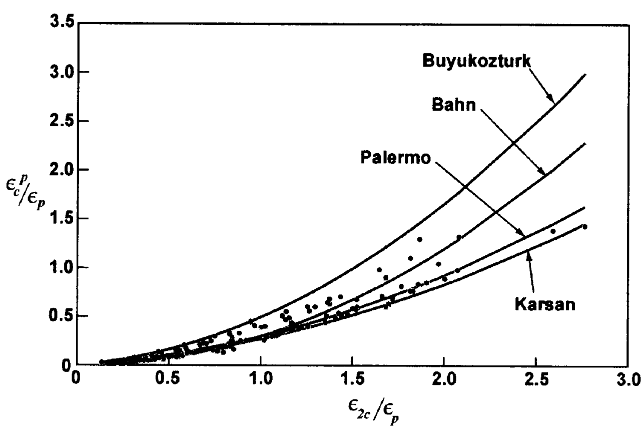

Fig. 2—Plastic offset models for concrete in compression.

relatively small plastic offsets, predicts more pinching in the hysteresis behavior of the concrete. This pinching phenomenon has been observed by Palermo and Vecchio[8] and Pilakoutas and Elnashai[16] in the load-deformation response of structural walls dominated by shear-related mechanisms.

In analysis, the plastic offset strain remains unchanged unless the previous maximum strain in the history of loading is exceeded.

The unloading response of concrete, in its simplest form, can be represented by a linear expression extending from the unloading strain to the plastic offset strain. This type of representation, however, is deficient in capturing the energy dissipated during an unloading/reloading cycle in compression. Test data of concrete under cyclic loading confirm that the unloading branch is nonlinear. To derive an expression to describe the unloading branch of concrete, a Ramberg-Osgood formulation similar to that used by Seckin[17] was adopted. The formulation is strongly influenced by the unloading and plastic offset strains. The general form of the unloading branch of the proposed model is expressed as

$$f_c(\Delta\varepsilon) = A + B\Delta\varepsilon + C\Delta\varepsilon^N \quad (2)$$

where f_c is the stress in the concrete on the unloading curve, and $\Delta\varepsilon$ is the strain increment, measured from the instantaneous strain on the unloading path to the unloading strain, A, B, and C are parameters used to define the general shape of the curve, and N is the Ramberg-Osgood power term. Applying boundary conditions from Fig. 1(a) and simplifying yields

$$f_c(\Delta\varepsilon) = f_{2c} + E_{c2}(\Delta\varepsilon) + \left[\frac{(E_{c3} - E_{c2})\Delta\varepsilon^N}{N(\varepsilon_c^p - \varepsilon_{2c})^{N-1}}\right] \quad (3)$$

where

$$\Delta\varepsilon = \varepsilon - \varepsilon_{2c} \quad (4)$$

and

$$N = \frac{(E_{c2} - E_{c3})(\varepsilon_c^p - \varepsilon_{2c})}{f_{c2} + E_{c2}(\varepsilon_c^p - \varepsilon_{2c})} \quad (5)$$

ε is the instantaneous strain in the concrete. The initial unloading stiffness E_{c2} is assigned a value equal to the initial tangent stiffness of the concrete E_c, and is routinely calculated as $2f_c'/\varepsilon_c'$. The unloading stiffness E_{c3}, which defines the stiffness at the end of the unloading phase, is defined as $0.071\ E_c$, and was adopted from Seckin.[17] f_{2c} is the stress calculated from the backbone curve at the peak unloading strain ε_{2c}.

Reloading can sufficiently be modeled by a linear response and is done so by most researchers. An important characteristic, however, which is commonly ignored, is the degradation in the reloading stiffness resulting from load cycling. Essentially, the reloading curve does not return to the backbone curve at the previous maximum unloading strain (refer to Fig. 1 (b)). Further straining is required for the reloading response to intersect the backbone curve. Mander, Priestley, and Park[6] attempted to incorporate this phenomenon by defining a new stress point on the reloading path that corresponded to the maximum unloading strain. The new stress point was assumed to be a function of the previous unloading stress and the stress at reloading reversal. Their approach, however, was stress-based and dependent on the backbone curve. The approach used herein is to define the reloading stiffness as a degrading function to account for the damage induced in the concrete due to load cycling. The degradation was observed to be a function of the strain recovery during unloading. The reloading response is then determined from

$$f_c = f_{ro} + E_{c1}(\varepsilon_c - \varepsilon_{ro}) \quad (6)$$

where f_c and ε_c are the stress and strain on the reloading path; f_{ro} is the stress in the concrete at reloading reversal and corresponds to a strain of ε_{ro}; and E_{c1} is the reloading stiffness, calculated as follows

$$E_{c1} = \frac{(\beta_d \cdot f_{max}) - f_{ro}}{\varepsilon_{2c} - \varepsilon_{ro}} \quad (7)$$

where

$$\beta_d = \frac{1}{1 + 0.10(\varepsilon_{rec}/\varepsilon_p)^{0.5}} \quad \text{for } |\varepsilon_c| < |\varepsilon_p| \quad (8)$$

and

$$\beta_d = \frac{1}{1 + 0.175(\varepsilon_{rec}/\varepsilon_p)^{0.6}} \quad \text{for } |\varepsilon_c| > |\varepsilon_p| \quad (9)$$

and

$$\varepsilon_{rec} = \varepsilon_{max} - \varepsilon_{min} \quad (10)$$

β_d is a damage indicator, f_{max} is the maximum stress in the concrete for the current unloading loop, and ε_{rec} is the amount of strain recovered in the unloading process and is the difference between the maximum strain ε_{max} and the minimum strain ε_{min} for the current hysteresis loop. The minimum strain is limited by the compressive plastic offset strain. The damage indicator was derived from test data on plain concrete from four series of tests: Buyukozturk and Tseng,[15] Bahn and Hsu,[14] Karsan and Jirsa,[13] and Yankelevsky and Reinhardt.[18] A total of 31 datum points were collected for the prepeak range (Fig. 3(a)) and 33 datum points for the postpeak regime (Fig. 3(b)). Because there was a negligible amount of scatter among the test series, the datum points were combined to formulate the model. Figure 3(a) and (b) illustrate good correlation with experimental data, indicating the link between the strain recovery and the damage due to load cycling. β_d is calculated for the first unloading/reloading cycle and retained until the previous maximum unloading strain is attained or exceeded. Therefore, no additional damage is induced in the concrete for hysteresis loops occurring at strains less than the maximum unloading strain. This phenomenon is further illustrated through the partial unloading and partial reloading formulations.

It is common for cyclic models in the literature to ignore the behavior of concrete for the case of partial unloading/ reloading. Some models establish rules for partial loadings from the full unloading/reloading curves. Other models explicitly consider the case of partial unloading followed by reloading to either the backbone curve or strains in excess of the previous maximum unloading strain. There exists, however, a lack of information considering the case where partial unloading is followed by partial reloading to strains less than the previous maximum unloading strain. This more general case was modeled using the experimental results of Bahn and Hsu.[14] The proposed rule for the partial unloading response is identical to that assumed for full unloading; however, the previous maximum unloading strain and corresponding stress are replaced by a variable unloading strain and stress, respectively. The unloading path is defined by the unloading stress and strain and the plastic offset strain, which remains unchanged unless the previous maximum strain is exceeded. For the case of partial unloading followed by reloading to a strain in excess of the previous maximum unloading strain, the reloading path is defined by the expressions governing full reloading. The case where concrete is partially unloaded and partially reloaded to a strain less than the previous maximum unloading strain is illustrated in Fig 4.

Five loading branches are required to construct the response of Fig. 4. Unloading Curve 1 represents full unloading from the maximum unloading strain to the plastic offset and is calculated from Eq. (3) to (5) for full unloading. Curve 2 defines reloading from the plastic offset strain and is defined by Eq. (6) to (10). Curve 3 represents the case of partial unloading from a reloading path at a strain less than the previous maximum unloading strain. The expressions used for full unloading are applied, with the exception of substituting the unloading stress and strain for the current hysteresis loop for the unloading stress and strain at the previous maximum unloading point. Curve 4 describes partial reloading from a partial unloading branch. The response follows a linear path from the load reversal point to the previous unloading point and assumes that damage is not accumulated in loops forming at strains less than the previous maximum unloading strain. This implies that the reloading stiffness of Curve 4 is greater than the reloading stiffness of Curve 2 and is consistent with test data reported by Bahn and Hsu.[14] The reloading stiffness for Curve 4 is represented by the following expression

$$E_{c1} = \frac{f_{max} - f_{ro}}{\varepsilon_{max} - \varepsilon_{ro}} \quad (11)$$

The reloading stress is then calculated using Eq. (6) for full reloading.

In further straining beyond the intersection with Curve 2, the response of Curve 4 follows the reloading path of Curve 5. The latter retains the damage induced in the concrete from the first unloading phase, and the stiffness is calculated as

$$E_{c1} = \frac{\beta_d \cdot f_{2c} - f_{max}}{\varepsilon_{2c} - \varepsilon_{max}} \quad (12)$$

The reloading stresses are then determined from the following

$$f_c = f_{max} + E_{c1}(\varepsilon_c - \varepsilon_{max}) \quad (13)$$

The proposed constitutive relations for concrete subjected to compressive cyclic loading are tested in Fig. 5 against the experimental results of Karsan and Jirsa.[13] The Palermo model generally captures the behavior of concrete under cyclic compressive loading. The nonlinear unloading and linear loading formulations agree well with the data, and the plastic offset strains are well predicted. It is apparent, though, that the reloading curves become nonlinear beyond the point of intersection with the unloading curves, often referred to as the

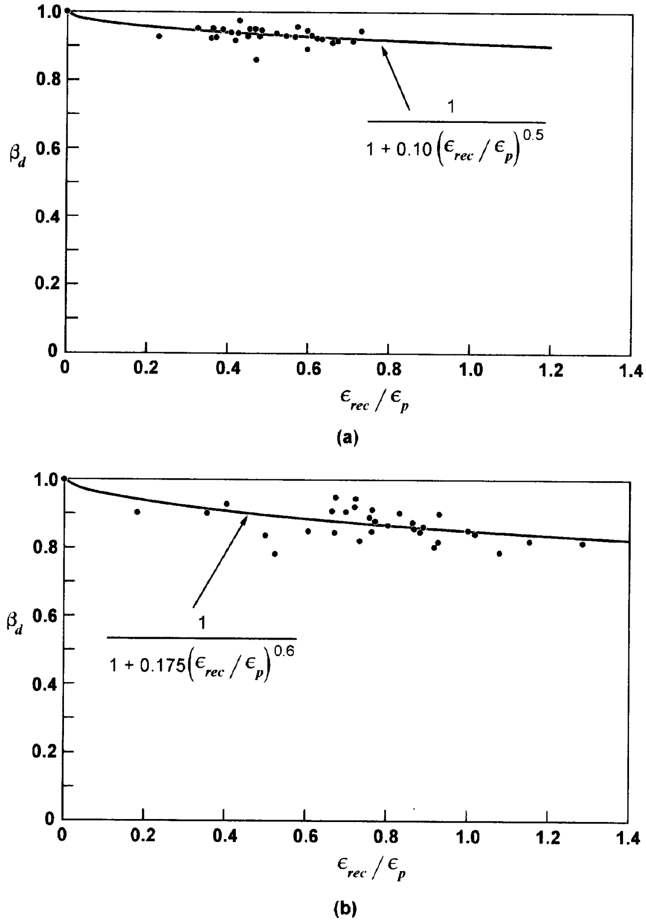

Fig. 3—Damage indicator for concrete in compression: (a) prepeak regime; and (b) postpeak regime.

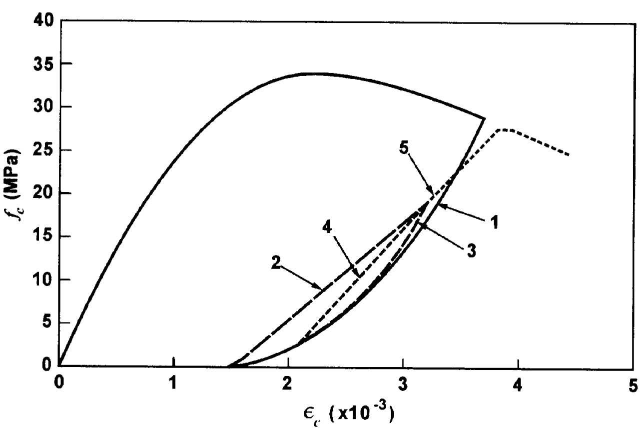

Fig. 4—Partial unloading/reloading for concrete in compression.

common point. The Palermo model can be easily modified to account for this phenomenon; however, unusually small load steps would be required in a finite element analysis to capture this behavior, and it was thus ignored in the model. Furthermore, the results tend to underestimate the intersection of the reloading path with the backbone curve. This is a direct result of the postpeak response of the concrete and demonstrates the importance of proper modeling of the postpeak behavior.

Tension response

Much less attention has been directed towards the modeling of concrete under cyclic tensile loading. Some researchers consider little or no excursions into the tension stress regime and those who have proposed models assume, for the most part, linear unloading/reloading responses with no plastic offsets. The latter was the approach used by Vecchio[5] in formulating a preliminary tension model. Stevens, Uzumeri, and Collins[19] reported a nonlinear response based on defining the stiffness along the unloading path; however, the models were verified with limited success. Okumura and Maekawa[2] proposed a hysteretic model for cyclic tension, in which a nonlinear unloading curve considered stresses through bond action and through closing of cracks. A linear reloading path was also assumed. Hordijk[20] used a fracture mechanics approach to formulate nonlinear unloading/reloading rules in terms of applied stress and crack opening displacements.

The proposed tension model follows the philosophy used to model concrete under cyclic compression loadings. Figure 6 (a) and (b) illustrate the unloading and reloading responses, respectively. The backbone curve, which assumes the monotonic behavior, consists of two parts adopted from the Modified Compression Field Theory[12]: that describing the precracked response and that representing postcracking tension-stiffened response.

A shortcoming of the current body of data is the lack of theoretical models defining a plastic offset for concrete in tension. The offsets occur when cracked surfaces come into contact during unloading and do not realign due to shear slip along the cracked surfaces. Test results from Yankelevsky and Reinhardt[21] and Gopalaratnam and Shah[22] provide data that can be used to formulate a plastic offset model (refer to Fig. 7). The researchers were able to capture the softening behavior of concrete beyond cracking in displacement-controlled testing machines. The plastic offset strain, in the proposed tension model, is used to define the shape of the unloading curve, the slope and damage of the reloading path, and the point at which cracked surfaces come into contact. Similar to concrete in compression, the offsets in tension seem to be dependent on the unloading strain from the backbone curve. The proposed offset model is expressed as

$$\varepsilon_c^p = 146\varepsilon_{1c}^2 + 0.523\varepsilon_{1c} \qquad (14)$$

where ε_c^p is the tensile plastic offset, and ε_{1c} is the unloading strain from the backbone curve. Figure 7 illustrates very good correlation to experimental data.

Observations of test data suggest that the unloading response of concrete subjected to tensile loading is nonlinear. The accepted approach has been to model the unloading branch as linear and to ignore the hysteretic behavior in the concrete

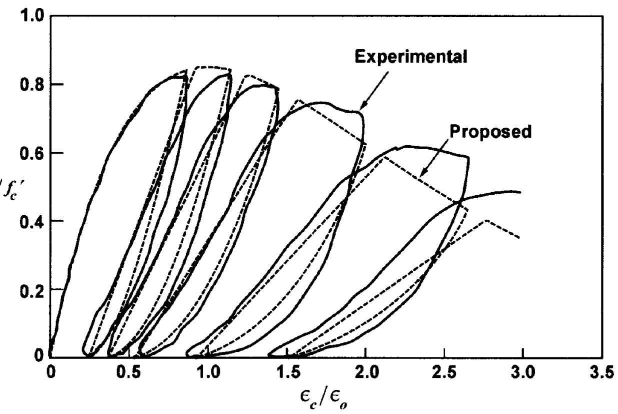

Fig. 5—Predicted response for cycles in compression.

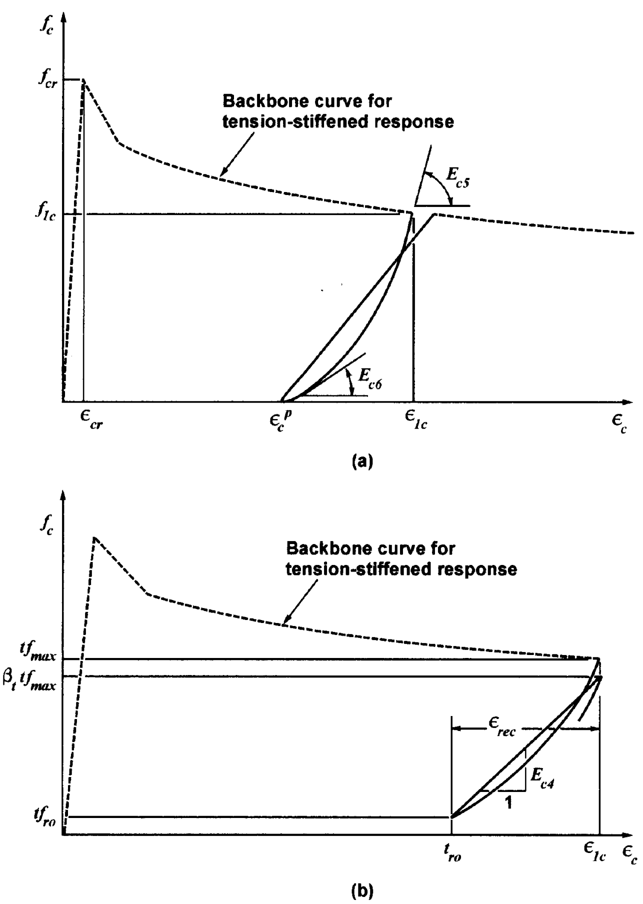

Fig. 6—Hysteresis models for concrete in tension: (a) unloading; and (b) reloading.

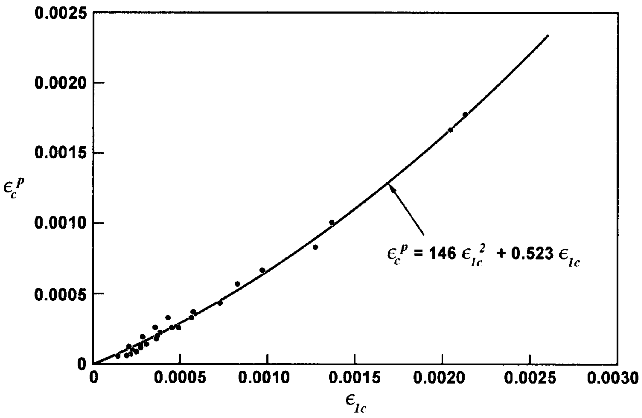

Fig. 7—Plastic offset model for concrete in tension.

due to cycles in tension. The approach used herein was to formulate a nonlinear expression for the concrete that would generate realistic hysteresis loops. To derive a model consistent with the compression field approach, a Ramberg-Osgood formulation, similar to that used for concrete in compression, was adopted and is expressed as

$$f_c = D + F\Delta\varepsilon + G\Delta\varepsilon^N \quad (15)$$

where f_c is the tensile stress in the concrete; $\Delta\varepsilon$ is the strain increment measured from the instantaneous strain on the unloading path to the unloading strain; D, F, and G are parameters that define the shape of the unloading curve; and N is a power term that describes the degree of nonlinearity.

Applying the boundary conditions from Fig. 6(a) and simplifying yields

$$f_c(\Delta\varepsilon) = f_{1c} - E_{c5}(\Delta\varepsilon) + \left[\frac{(E_{c5} - E_{c6})\Delta\varepsilon^N}{N(\varepsilon_{1c} - \varepsilon_c^p)^{N-1}}\right] \quad (16)$$

where

$$\Delta\varepsilon = \varepsilon_{1c} - \varepsilon \quad (17)$$

and

$$N = \frac{(E_{c5} - E_{c6})(\varepsilon_{1c} - \varepsilon_c^p)}{E_{c5}(\varepsilon_{1c} - \varepsilon_c^p) - f_{1c}} \quad (18)$$

f_{1c} is the unloading stress from the backbone curve, and E_{c5} is the initial unloading stiffness, assigned a value equal to the initial tangent stiffness E_c. The unloading stiffness E_{c6}, which defines the stiffness at the end of the unloading phase, was determined from unloading data reported by Yankelevsky and Reinhardt.[21] By varying the unloading stiffness E_{c6}, the following models were found to agree well with test data

$$E_{c6} = 0.071 \cdot E_c(0.001/\varepsilon_{1c}) \qquad \varepsilon_{1c} \le 0.001 \quad (19)$$

$$E_{c6} = 0.053 \cdot E_c(0.001/\varepsilon_{1c}) \qquad \varepsilon_{1c} > 0.001 \quad (20)$$

The Okamura and Maekawa[2] model tends to overestimate the unloading stresses for plain concrete, owing partly to the fact that the formulation is independent of a tensile plastic offset strain. The formulations are a function of the unloading point and a residual stress at the end of the unloading phase. The residual stress is dependent on the initial tangent stiffness and the strain at the onset of unloading. The linear unloading response suggested by Vecchio[5] is a simple representation of the behavior but does not capture the nonlinear nature of the concrete and underestimates the energy dissipation. The proposed model captures the nonlinear behavior and energy dissipation of the concrete.

The state of the art in modeling reloading of concrete in tension is based on a linear representation, as described by, among others, Vecchio[5] and Okamura and Maekawa.[2] The response is assumed to return to the backbone curve at the previous unloading strain and ignores damage induced to the concrete due to load cycling. Limited test data confirm that linear reloading sufficiently captures the general response of the concrete; however, it is evident that the reloading stiffness accumulates damage as the unloading strain increases. The approach suggested herein is to model the reloading behavior as linear and to account for a degrading reloading stiffness. The latter is assumed to be a function of the strain recovered during the unloading phase and is illustrated in Fig. 8 against data reported by Yankelevsky and Reinhardt.[21] The reloading stress is calculated from the following expression

$$f_c = \beta_t \cdot tf_{max} - E_{c4}(\varepsilon_{1c} - \varepsilon_c) \quad (21)$$

where

$$E_{c4} = \frac{(\beta_t \cdot tf_{max}) - tf_{ro}}{\varepsilon_{1c} - t_{ro}} \quad (22)$$

f_c is the tensile stress on the reloading curve and corresponds to a strain of ε_c. E_{c4} is the reloading stiffness, β_t is a tensile damage indicator, tf_{max} is the unloading stress for the current hysteresis loop, and tf_{ro} is the stress in the concrete at reloading reversal corresponding to a strain of t_{ro}. The damage parameter β_t is calculated from the following relation

$$\beta_t = \frac{1}{1 + 1.15(\varepsilon_{rec})^{0.25}} \quad (23)$$

where

$$\varepsilon_{rec} = \varepsilon_{max} - \varepsilon_{min} \quad (24)$$

ε_{rec} is the strain recovered during an unloading phase. It is the difference between the unloading strain ε_{max} and the minimum strain at the onset of reloading ε_{min}, which is limited by the plastic offset strain. Figure 8 depicts good correlation between the proposed formulation and the limited experimental data.

Following the philosophy for concrete in compression, β_t is calculated for the first unloading/reloading phase and retained until the previous maximum strain is at least attained.

The literature is further deficient in the matter of partial unloading followed by partial reloading in the tension stress regime. Proposed herein is a partial unloading/reloading

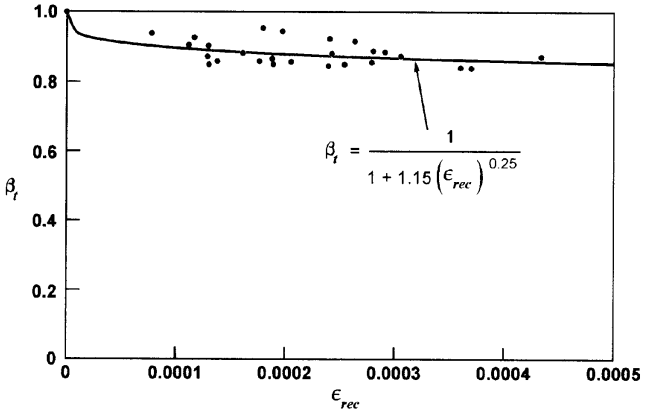

Fig. 8—Damage model for concrete in tension.

model that directly follows the rules established for concrete in compression. No data exist, however, to corroborate the model. Figure 9 depicts the proposed rules for a concrete element, lightly reinforced to allow for a post-cracking response.

Curve 1 corresponds to a full unloading response and is identical to that assumed by Eq. (16) to (18). Reloading from a full unloading curve is represented by Curve 2 and is computed from Eq. (21) to (24). Curve 3 represents the case of partial unloading from a reloading path at a strain less than the previous maximum unloading strain. The expressions for full unloading are used; however, the strain and stress at unloading, now variables, replace the strain and stress at the previous peak unloading point on the backbone curve. Reloading from a partial unloading segment is described by Curve 4. The response follows a linear path from the reloading strain to the previous unloading strain. The model explicitly assumes that damage does not accumulate for loops that form at strains less than the previous maximum unloading strain in the history of loading. Therefore, the reloading stiffness of Curve 4 is larger than the reloading stiffness for the first unloading/reloading response of Curve 2. The partial reloading stiffness, defining Curve 4, is calculated by the following expression

$$E_{c4} = \frac{tf_{max} - tf_{ro}}{\varepsilon_{max} - t_{ro}} \tag{25}$$

and the reloading stress is then determined from

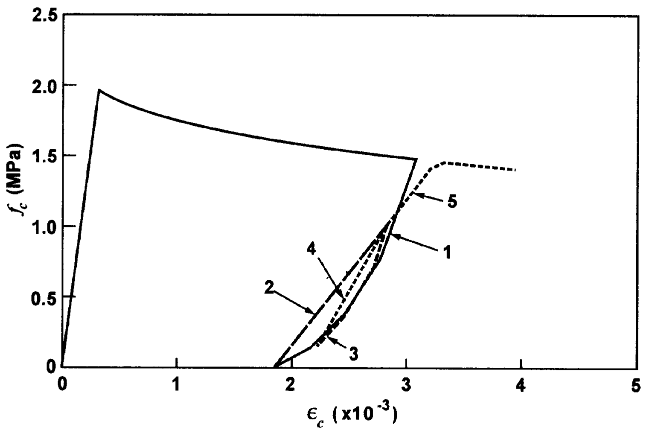

Fig. 9—Partial unloading/reloading for concrete in tension.

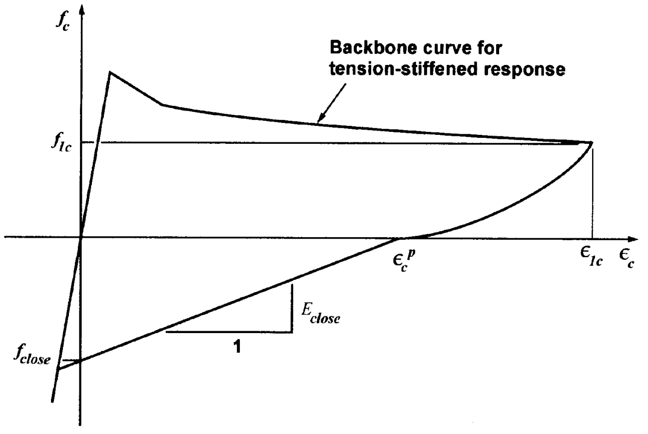

Fig. 10—Crack-closing model.

$$f_c = tf_{ro} + E_{c4}(\varepsilon_c - t_{ro}) \tag{26}$$

As loading continues along the reloading path of Curve 4, a change in the reloading path occurs at the intersection with Curve 2. Beyond the intersection, the reloading response follows the response of Curve 5 and retains the damage induced to the concrete from the first unloading/reloading phase. The stiffness is then calculated as

$$E_{c4} = \frac{\beta_t \cdot f_{1c} - tf_{max}}{\varepsilon_{1c} - \varepsilon_{max}} \tag{27}$$

The reloading stresses can then be calculated according to

$$f_c = tf_{max} + E_{c4}(\varepsilon_c - \varepsilon_{max}) \tag{28}$$

The previous formulations for concrete in tension are preliminary and require experimental data to corroborate. The models are, however, based on realistic assumptions derived from the models suggested for concrete in compression.

CRACK-CLOSING MODEL

In an excursion returning from the tensile domain, compressive stresses do not remain at zero until the cracks completely close. Compressive stresses will arise once cracked surfaces come into contact. The recontact strain is a function of factors such as crack-shear slip. There exists limited data to form an accurate model for crack closing, and the preliminary model suggested herein is based on the formulations and assumptions suggested by Okamura and Maekawa.[2] Figure 10 is a schematic of the proposed model.

The recontact strain is assumed equal to the plastic offset strain for concrete in tension. The stiffness of the concrete during closing of cracks, after the two cracked surfaces have come into contact and before the cracks completely close, is smaller than that of crack-free concrete. Once the cracks completely close, the stiffness assumes the initial tangent stiffness value. The crack-closing stiffness E_{close} is calculated from

$$E_{close} = \frac{f_{close}}{\varepsilon_c^p} \tag{29}$$

where

$$f_{close} = -E_c(0.0016 \cdot \varepsilon_{1c} + 50 \times 10^{-6}) \tag{30}$$

f_{close}, the stress imposed on the concrete as cracked surfaces come into contact, consists of two terms taken from the Okamura and Maekawa[2] model for concrete in tension. The first term represents a residual stress at the completion of unloading due to stress transferred due to bond action. The second term represents the stress directly related to closing of cracks. The stress on the closing-of-cracks path is then determined from the following expression

$$f_c = E_{close}(\varepsilon_c - \varepsilon_c^p) \tag{31}$$

After the cracks have completely closed and loading continues into the compression strain region, the reloading rules for concrete in compression are applicable, with the stress in the concrete at the reloading reversal point assuming a value of f_{close}.

For reloading from the closing-of-cracks curve into the tensile strain region, the stress in the concrete is assumed to be linear, following the reloading path previously established for tensile reloading of concrete.

In lieu of implementing a crack-closing model, plastic offsets in tension can be omitted, and the unloading stiffness at the completion of unloading E_{c6} can be modified to ensure that the energy dissipation during unloading is properly captured. Using data reported by Yankelevsky and Reinhardt,[21] a formulation was derived for the unloading stiffness at zero loads and is proposed as a function of the unloading strain on the backbone curve as follows

$$E_{c6} = -1.1364(\varepsilon_{1c}^{-0.991}) \quad (32)$$

Implicit in the latter model is the assumption that, in an unloading excursion in the tensile strain region, the compressive stresses remain zero until the cracks completely close.

REINFORCEMENT MODEL

The suggested reinforcement model is that reported by Vecchio,[5] and is illustrated in Fig. 11. The monotonic response of the reinforcement is assumed to be trilinear. The initial response is linear elastic, followed by a yield plateau, and ending with a strain-hardening portion. The hysteretic response of the reinforcement has been modeled after Seckin,[17] and the Bauschinger effect is represented by a Ramberg-Osgood formulation.

The monotonic response curve is assumed to represent the backbone curve. The unloading portion of the response follows a linear path and is given by

$$f_s(\varepsilon_i) = f_{s-1} + E_r(\varepsilon_i - \varepsilon_{s-1}) \quad (33)$$

where $f_s(\varepsilon_i)$ is the stress at the current strain of ε_i, f_{s-1} and ε_{s-1} are the stress and strain from the previous load step, and E_r is the unloading modulus and is calculated as

$$E_r = E_s \qquad \text{if } (\varepsilon_m - \varepsilon_o) < \varepsilon_y \quad (34)$$

$$E_r = E_s\left(1.05 - 0.05\frac{\varepsilon_m - \varepsilon_o}{\varepsilon_y}\right) \text{ if } \varepsilon_y < (\varepsilon_m - \varepsilon_o) < 4\varepsilon_y \quad (35)$$

$$E_r = 0.85E_s \text{ if } (\varepsilon_m - \varepsilon_o) > 4\varepsilon_y \quad (36)$$

where E_s is the initial tangent stiffness; ε_m is the maximum strain attained during previous cycles; ε_o is the plastic offset strain; and ε_y is the yield strain.

The stresses experienced during the reloading phase are determined from

$$f_s(\varepsilon_i) = E_r(\varepsilon_i - \varepsilon_o) + \frac{E_m - E_r}{N \cdot (\varepsilon_m - \varepsilon_o)^{N-1}} \cdot (\varepsilon_i - \varepsilon_o)^N \quad (37)$$

where

$$N = \frac{(E_m - E_r)(\varepsilon_m - \varepsilon_o)}{f_m - E_r(\varepsilon_m - \varepsilon_o)} \quad (38)$$

f_m is the stress corresponding to the maximum strain recorded during previous loading; and E_m is the tangent stiffness at ε_m.

The same formulations apply for reinforcement in tension or compression. For the first reverse cycle, ε_m is taken as zero and $f_m = f_y$, the yield stress.

IMPLEMENTATION AND VERIFICATION

The proposed formulations for concrete subjected to reversed cyclic loading have been implemented into a two-dimensional nonlinear finite element program, which was developed at the University of Toronto.[23]

The program is applicable to concrete membrane structures and is based on a secant stiffness formulation using a total-load, iterative procedure, assuming smeared rotating cracks. The package employs the compatibility, equilibrium, and constitutive relations of the Modified Compression Field Theory.[12] The reinforcement is typically modeled as smeared within the element but can also be discretely represented by truss-bar elements.

The program was initially restricted to conditions of monotonic loading, and later developed to account for material prestrains, thermal loads, and expansion and confinement effects. The ability to account for material prestrains provided the framework for the analysis capability of reversed cyclic loading conditions.[5]

For cyclic loading, the secant stiffness procedure separates the total concrete strain into two components: an elastic strain and a plastic offset strain. The elastic strain is used to compute an effective secant stiffness for the concrete, and, therefore, the plastic offset strain must be treated as a strain offset, similar to an elastic offset as reported by Vecchio.[4] The plastic offsets in the principal directions are resolved into components relative to the reference axes. From the prestrains, free joint displacements are determined as functions of the element geometry. Then, plastic prestrain nodal forces can be evaluated using the effective element stiffness matrix due to the concrete component. The plastic offsets developed in

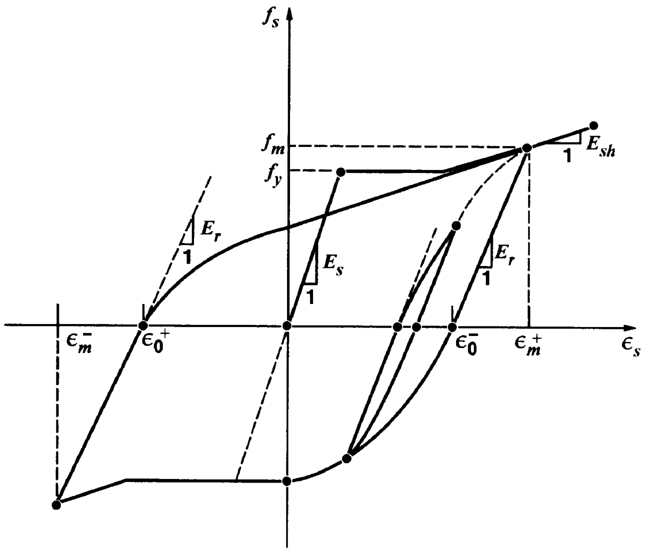

Fig. 11—Hysteresis model for reinforcement, adapted from Seckin (1981).

each of the reinforcement components are also handled in a similar manner.

The total nodal forces for the element, arising from plastic offsets, are calculated as the sum of the concrete and reinforcement contributions. These are added to prestrain forces arising from elastic prestrain effects and nonlinear expansion effects. The finite element solution then proceeds.

The proposed hysteresis rules for concrete in this procedure require knowledge of the previous strains attained in the history of loading, including, amongst others: the plastic offset strain, the previous unloading strain, and the strain at reloading reversal. In the rotating crack assumption, the principal strain directions may be rotating presenting a complication. A simple and effective method of tracking and defining the strains is the construction of Mohr's circle. Further details of the procedure used for reversed cyclic loading can be found from Vecchio.[5]

A comprehensive study, aimed at verifying the proposed cyclic models using nonlinear finite element analyses, will be presented in a companion paper.[9] Structures considered will include shear panels and structural walls available in the literature, demonstrating the applicability of the proposed formulations and the effectiveness of a secant stiffness-based algorithm employing the smeared crack approach. The structural walls will consist of slender walls, with height-width ratios greater than 2.0, which are heavily influenced by flexural mechanisms, and squat walls where the response is dominated by shear-related mechanisms. The former is generally not adequate to corroborate constitutive formulations for concrete.

CONCLUSIONS

A unified approach to constitutive modeling of reversed cyclic loading of reinforced concrete has been presented.

The constitutive relations for concrete have been formulated in the context of a smeared rotating crack model, consistent with a compression field approach. The models are intended for a secant stiffness-based algorithm but are also easily adaptable in programs assuming either fixed cracks or fixed principal stress directions.

The concrete cyclic models consider concrete in compression and concrete in tension. The unloading and reloading rules are linked to backbone curves, which are represented by the monotonic response curves. The backbone curves are adjusted for compressive softening and confinement in the compression regime, and for tension stiffening and tension softening in the tensile region.

Unloading is assumed nonlinear and is modeled using a Ramberg-Osgood formulation, which considers boundary conditions at the onset of unloading and at zero stress. Unloading, in the case of full loading, terminates at the plastic offset strain. Models for the compressive and tensile plastic offset strains have been formulated as a function of the maximum unloading strain in the history of loading.

Reloading is modeled as linear with a degrading reloading stiffness. The reloading response does not return to the backbone curve at the previous unloading strain, and further straining is required to intersect the backbone curve. The degrading reloading stiffness is a function of the strain recovered during unloading and is bounded by the maximum unloading strain and the plastic offset strain.

The models also consider the general case of partial unloading and partial reloading in the region below the previous maximum unloading strain.

NOTATION

E_c = initial modulus of concrete
E_{close} = crack-closing stiffness modulus of concrete in tension
E_{c1} = compressive reloading stiffness of concrete
E_{c2} = initial unloading stiffness of concrete in compression
E_{c3} = compressive unloading stiffness at zero stress in concrete
E_{c4} = reloading stiffness modulus of concrete in tension
E_{c5} = initial unloading stiffness modulus of concrete in tension
E_{c6} = unloading stiffness modulus at zero stress for concrete in tension
E_m = tangent stiffness of reinforcement at previous maximum strain
E_r = unloading stiffness of reinforcement
E_s = initial modulus of reinforcement
E_{sh} = strain-hardening modulus of reinforcement
f_{1c} = unloading stress from backbone curve for concrete in tension
f_{2c} = unloading stress on backbone curve for concrete in compression
f_c = normal stress of concrete
f'_c = peak compressive strength of concrete cylinder
f_{close} = crack-closing stress for concrete in tension
f_{cr} = cracking stress of concrete in tension
f_m = reinforcement stress corresponding to maximum strain in history
f_{max} = maximum compressive stress of concrete for current unloading cycle
f_p = peak principal compressive stress of concrete
f_{ro} = compressive stress at onset of reloading in concrete
f_s = average stress for reinforcement
f_{s-1} = stress in reinforcement from previous load step
f_y = yield stress for reinforcement
tf_{max} = maximum tensile stress of concrete for current unloading cycle
tf_{ro} = tensile stress of concrete at onset of reloading
t_{ro} = tensile strain of concrete at onset of reloading
β_d = damage indicator for concrete in compression
β_t = damage indicator for concrete in tension
$\Delta\varepsilon$ = strain increment on unloading curve in concrete
ε = instantaneous strain in concrete
ε_0 = plastic offset strain of reinforcement
ε_{1c} = unloading strain on backbone curve for concrete in tension
ε_{2c} = compressive unloading strain on backbone curve of concrete
ε_c = compressive strain of concrete
ε'_c = strain at peak compressive stress in concrete cylinder
ε_c^p = residual (plastic offset) strain of concrete
ε_{cr} = cracking strain for concrete in tension
ε_i, ε_s = current stress of reinforcement
ε_m = maximum strain of reinforcement from previous cycles
ε_{max} = maximum strain for current cycle
ε_{min} = minimum strain for current cycle
ε_p = strain corresponding to maximum concrete compressive stress
ε_{rec} = strain recovered during unloading in concrete
ε_{ro} = compressive strain at onset of reloading in concrete
ε_{sh} = strain of reinforcement at which strain hardening begins
ε_{s-1} = strain of reinforcement from previous load step
ε_y = yield strain of reinforcement

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